SPECIFIC FEATURES OF THE PROPAGATION OF NONSTATIONARY SHOCK WAVES IN A BUBBLY FLUID WITH A NON-NEWTONIAN CARRIER PHASE

A. A. Gubaidullin, O. Sh. Rustyumova, and S. A. Bekishev   UDC 532.529

The wave dynamics of bubbly media with viscous and inviscid Newtonian carrier fluid has been thus far studied fairly well [1–4]. At the same time, wavy motions of such two-phase systems based on Newtonian fluids with complex rheology (polymer solutions and melts, suspensions, paraffin-base and tarry oils, etc.) have been studied rather poorly, despite their wide application in practice. Among those studied are some problems of the behavior of an individual bubble in a viscoelastic relaxing polymer fluid and the propagation of acoustic waves in a similar liquid with bubbles [5, 6]. These problems have been most extensively elucidated by Levitskii and Shul'man [6]. The problems of dynamics of nonstationary nonlinear waves still remain unstudied.

In the present paper, we consider the specific features of the propagation of nonstationary shock waves in a non-Newtonian fluid with gas bubbles. In this connection, it is of interest to compare the wave behavior in bubbly mixtures with non-Newtonian and viscous Newtonian carrier phases and also to analyze the effect of the governing parameters of a two-phase mixture (initial viscosity and density of fluids, volume concentration of bubbles and their sizes, kind of gas, time of stress relaxation, etc.).

We present a system of equations that describes the dynamic behavior of the indicated two-phase fluid with allowance for the following assumptions [1]: the bubble sizes are considerably greater than the molecular-kinetic ones and are considerably smaller than the distances at which the averaged or macroscopic parameters of the mixture change significantly; the mixture is monodisperse; a direct interaction between bubbles can be ignored; bubble fragmentation and coagulation and the formation of new bubbles are absent; the gas pressure inside bubbles is uniform; the velocities of the macroscopic phase motion coincide; and the temperature of the carrier fluid and its density are constant.

The following differential equations of conservation of phase masses, the mass of an individual bubble and of the momentum of a mixture under the above assumptions for nonstationary plane one-dimensional motion are valid:

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_1 v}{\partial x} = 0, \qquad \frac{\partial \rho_2}{\partial t} + \frac{\partial \rho_2 v}{\partial x} = 0, \qquad \frac{d}{dt}\left(\rho_2^0 a^3\right) = 0,$$

$$\rho \frac{dv}{dt} + \frac{\partial p}{\partial x} = 0 \qquad \left(\frac{d}{dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right).$$

Here and below, ρ, p, and v are the density, pressure, and velocity of the mixture; ρ_i^0, ρ_i, α_i, T_i, and p_i are the real and reduced densities, volume concentration, temperature, and pressure of the ith phase; the subscripts 1 and 2 denote the parameters of the fluid and the gas, respectively; the superscript 0 denotes the real value of the parameter; a is the radius of a bubble; w is the radial velocity of the bubble boundary; and n is the number of bubbles per unit volume.

The equations of heat influx to the phases are of the form

$$\rho_2 c_{V2} \frac{dT_2}{dt} = \frac{\alpha_2 p_2}{\rho_2^0} \frac{d\rho_2^0}{dt} + n q_2, \qquad T_1 = \text{const},$$

where q_2 is the heat influx from the fluid per bubble. Within the framework of a two-temperature scheme [1], the intensity of interphase heat transfer is specified by the relation

$$q_2 = 4\pi a^3 \frac{\lambda_2 \mathrm{Nu}_2}{2a} (T_1 - T_2),$$

where λ_2 is the heat-conduction coefficient for the gas and Nu_2 is the Nusselt number. The parameter Nu_2 can be given [2] in the form

$$\mathrm{Nu}_2 = \begin{cases} 10, & \mathrm{Pe}_2 \leqslant 100, \\ \sqrt{\mathrm{Pe}_2}, & \mathrm{Pe}_2 > 100, \end{cases} \qquad \mathrm{Pe}_2 = 12(\gamma_2 - 1) \frac{T_1}{|T_1 - T_2|} \frac{a|w|}{\nu_2^{(T)}}, \qquad \nu_2^{(T)} = \frac{\lambda_2}{\rho_2^0 c_{p2}}.$$

Here γ_2 is the adiabatic exponent, $\nu_2^{(T)}$ is the thermal diffusivity, c_{V2} and c_{p2} are the specific heats of the gas for a constant volume and pressure, and Pe_2 is the Peclet number.

The gas is assumed to be ideal and calorically perfect, and the fluid is assumed to be incompressible: $p_2 = \rho_2^0(\gamma_2 - 1) c_{V2} T_2$, $u_2 = c_{V2} T_2$, and $\rho_1^0 = \mathrm{const}$.

By definition, $\rho_1 = \alpha_1 \rho_1^0$, $\rho_2 = \alpha_2 \rho_2^0$, $\rho = \rho_1 + \rho_2$, $\alpha_1 + \alpha_2 = 1$, $\alpha_2 = (4/3)\pi a^3 n$, and $p = \alpha_1 p_1 + \alpha_2(p_2 - 2\Sigma/a)$ (Σ is the surface tension coefficient of the fluid).

The Rayleigh–Lamb equation of radial oscillations of a single bubble, which is generalized to the case of a non-Newtonian fluid, was derived in [6] and is of the following form in the spherical system of coordinates (r, φ, ψ):

$$\rho_1^0 \left(a \frac{dw}{dt} + \frac{3}{2} w^2 \right) + p_1 - p_2 + \frac{2\Sigma}{a} = 2 \int\limits_a^\infty \frac{\tau^{(rr)} - \tau^{(\varphi\varphi)}}{r} dr, \qquad w = \frac{da}{dt}, \tag{1}$$

where τ is the stress tensor, which is determined by the Maxwell-type rheologic equation of state with the upper convective derivative with a single relaxation time t_r [6–8]:

$$\tau = \tau_s + \tau_p, \qquad \tau_s = 2\eta_s \mathbf{e}, \qquad \eta_s + \eta_p = \eta_0, \qquad \tau_p + t_r \left[\frac{d\tau_p}{dt} - \frac{1}{2}(\tau_p \cdot \mathbf{e} + \mathbf{e} \cdot \tau_p) \right] = 2\eta_p \mathbf{e}.$$

Here $\mathbf{e}$ is the strain rate tensor, the point denotes the inner product of the second-rank tensors (product of the matrices); η_0, η_s, and η_p is the initial viscosity of the solution, the viscosity of the solvent, and the viscosity of the polymer grid in the solution; and the subscripts p and s refer to the polymer and the solvent, respectively.

We write the equations for the normal components $\tau^{(rr)}$ and $\tau^{(\varphi\varphi)}$ as follows [6]:

$$\tau_p^{(rr)} + t_r \left(\frac{d\tau_p^{(rr)}}{dt} + 2\tau_p^{(rr)} \frac{a^2 w}{r^3} \right) = -4\eta_p \frac{a^2 w}{r^3}, \qquad \tau_p^{(\varphi\varphi)} + t_r \left(\frac{d\tau_p^{(\varphi\varphi)}}{dt} - \tau_p^{(\varphi\varphi)} \frac{a^2 w}{r^3} \right) = 2\eta_p \frac{a^2 w}{r^3}; \tag{2}$$

$$\tau_s^{(rr)} = -4\eta_s \frac{a^2 w}{r^3}, \qquad \tau_s^{(\varphi\varphi)} = 2\eta_s \frac{a^2 w}{r^3}. \tag{3}$$

From (2), one can find $\tau^{(rr)}$ and $\tau^{(\varphi\varphi)}$ and, having substituted them into the right-hand side of Eq. (1), one can obtain [6]

$$\rho_1^0 \left(a \frac{dw}{dt} + \frac{3}{2} w^2 \right) + \frac{2\Sigma}{a} = p_2 - p_1 + S, \tag{4}$$

where

$$S = S_p + S_s; \tag{5}$$

$$S_s = -4\eta_s \frac{w}{a}; \tag{6}$$

$$\frac{dS_p}{dt} + \left(\frac{1}{t_r} + \frac{2w}{a} \right) S_p = -4 \frac{\eta_p}{t_r} \frac{w}{a}. \tag{7}$$

Comparison of (6) and (7) with (2) and (3) shows that for $r = a$, we have $S_p = \tau_p^{(rr)}$ and $S_s = \tau_s^{(rr)}$, i.e., the parameters S_p and S_s have the sense of stresses on the bubble surface in the polymer and in the solvent, respectively.

We analyze the limiting cases of Eq. (7). Let t_* be the typical time of bubble pulsations that determines the time scale of the process, and S_{*p} be the scale of stress pulsations in the polymer. We obtain the following estimates of the terms of Eq. (7):

$$\left|\frac{dS_p}{dt}\right| \sim \frac{S_{*p}}{t_*}, \qquad \left|\frac{1}{t_r}S_p\right| \sim \frac{S_{*p}}{t_r}, \tag{8}$$

$$\left|2\frac{w}{a}S_p\right| \sim \frac{S_{*p}}{t_*} \quad \left(\text{as} \quad a \sim a_0, w = \frac{da}{dt} \sim \frac{a_0}{t_*}\right), \quad \left|-4\frac{\eta_p}{t_r}\frac{w}{a}\right| \sim \frac{\eta_p}{t_r t_*}.$$

For relaxation times smaller than the typical time of pulsations ($t_r \ll t_*$), we can ignore the terms on the right-hand side of Eq. (7) that contain time derivatives, i.e., the first and third terms versus the second (S_p/t_r). After that, only terms that should be of the same order on both sides of Eq. (7) are left, i.e., $S_{*p} \sim \eta_p/t_*$. For stresses in the polymer, we write the following relation:

$$S_p \approx -4\eta_p w/a, \qquad t_r \ll t_*, \tag{9}$$

which, with allowance for (6), yields $S = S_p + S_s \approx -4\eta_0 w/a$ and $t_r \ll t_*$ for stresses in the solution, i.e., is of the same form as that for a Newtonian fluid with a viscosity equal to the initial viscosity of the polymer solution.

For relaxation times greater than the typical time of pulsations ($t_r \gg t_*$), it follows from the estimates (8) that we can ignore the term S_p/t_r on the left-hand side, versus the first and third terms with derivatives. We obtain the relation

$$\frac{dS_p}{dt} + 2\frac{w}{a}S_p = -4\frac{\eta_p}{t_r}\frac{w}{a}, \qquad t_r \gg t_*,$$

which, with allowance for the initial conditions ($t = 0$: $S_p = 0$ and $a = a_0$), takes the form

$$S_p = 2\frac{\eta_p}{t_r}\left[\left(\frac{a_0}{a}\right)^2 - 1\right], \qquad t_r \gg t_*. \tag{10}$$

This corresponds to the nonlinear elastic restoring force. With small deviations of the bubble radius from the initial one, it can be reduced to the form

$$S_p \approx -4\frac{\eta_p}{t_r}\frac{\Delta a}{a_0}, \qquad t_r \gg t_*, \quad \Delta a \ll a_0, \quad a = a_0 + \Delta a. \tag{11}$$

Relations (10) and (11) show that in this limiting case, the polymer grid in the solution behaves like an elastic medium whose elasticity modulus G determines the scale of stress pulsations $S_{*p} \sim \eta_p/t_r = G$. The total stress in the solution consists of the elastic component associated with the polymer and the viscous component associated with the solvent:

$$S = S_p + S_s \approx 2\frac{\eta_p}{t_r}\left[\left(\frac{a_0}{a}\right)^2 - 1\right] - 4\eta_s\frac{w}{a}, \qquad t_r \gg t_*. $$

Clearly, the elastic stress in the polymer will exert an effect if it is comparable to or greater than the gas elasticity in the bubble, which is determined by the pressure increment Δp_2 in the Rayleigh–Lamb equation. Let $p_2/p_{20} = (\rho_2^0/\rho_{20}^0)^\ae$, $a = a_0 + \Delta a$, and $p_2 = p_{20} + \Delta p_2$. Note that $\Delta a \ll a_0$ and $\Delta p_2 \ll p_{20}$. Then

$$\Delta p_2 \approx -3\ae p_{20}\Delta a/a_0 = -4G_g\Delta a/a_0, \qquad G_g = (3/4)\ae p_{20}. \tag{12}$$

The elastic stress in the polymer appears if either $|S_p| \geqslant |\Delta p_2|$ or $G \geqslant G_g$. It is worth noting that, as applied to the shock wave, we should use the equilibrium pressure value behind the wavefront p_{20} as the value of p_e in (12).

Based on the above analysis, we can assert that in both limiting cases, in shock waves the polymer carrier phase behaves like a Newtonian fluid with viscosities η_0 and η_s, respectively, i.e., we should expect that with variation in the relaxation times, shock waves have two limiting structures: a shock wave propagating as in a Newtonian bubbly fluid with a viscosity equal to the initial viscosity of the solution η_0 ($t_r \ll t_*$) and a
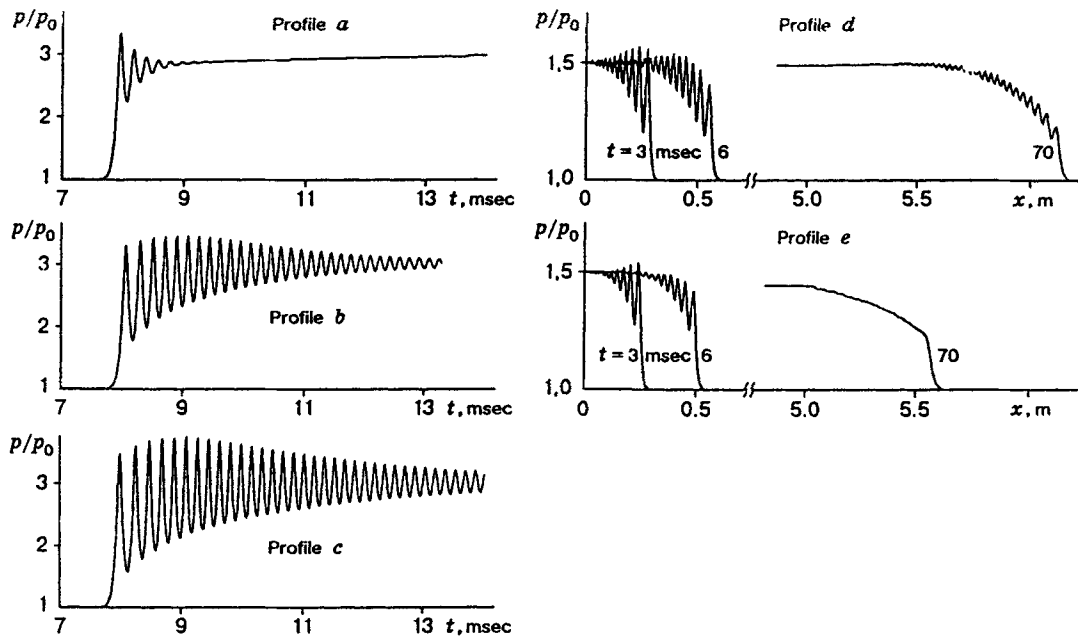
Fig. 1

shock wave propagating as in a Newtonian bubbly fluid with a viscosity equal to the viscosity of the solvent η_s ($t_r \gg t_*$). In both cases, one can model the polymer solution using a Newtonian fluid if the elasticity modulus of the polymer grid is smaller than the elasticity modulus of the gas in bubbles.

Figure 1 shows the effect of the relaxation time t_r on the shock-wave structure and also profiles (a)–(c) of the dimensionless pressure at the space point $x = 20$ cm for the relaxation time $t_r = 0.001, 01$, and 1 msec. In this case, the characteristic time is $t_* = 0.1$ msec. It is seen that it correlates with the time of cavity closing in the bubble $t_0 = a_0\sqrt{\rho_{10}^0/p_0}$. The mixture and wave parameters are as follows: $\rho_{10}^0 = 998$ kg/m^3, $a_0 = 1$ mm, $\eta_0 = 1.48$ Pa · sec, $\eta_s = 1$ mPa · sec, $\alpha_{20} = 0.02$, $T_0 = 293$ K, $p_0 = 0.1$ MPa, and $p_e = 0.3$ MPa. It follows from Fig. 1 that the oscillation amplitudes increase considerably with increasing relaxation time. A certain increase in the pulsation period is observed as well. Calculations have shown that for relaxation times smaller than 0.001 msec, the pressure profiles almost coincide with each other and with the corresponding curve for a Newtonian carrier fluid whose viscosity is equal to the initial viscosity of the polymer solution. In the other limiting case of large relaxation times ($t_r \geqslant 1$ msec), the pressure profiles coincide with each other and with the curve for a bubbly fluid in which the carrier fluid is of a constant viscosity equal to the solvent viscosity.

Modeling of wave processes in a non-Newtonian bubbly fluid is more complicated compared with a Newtonian fluid. In view of this, it is important to know whether it is possible to model, in the general case, the behavior of a non-Newtonian carrier phase using a Newtonian fluid with a chosen effective viscosity. Calculations have shown that this is impossible in the general case. In particular, even if at a certain moment of the evolution the wave shapes in a non-Newtonian bubbly fluid and in a specially chosen Newtonian bubbly fluid coincide, they are strongly different in these fluids at a later moment.

Let us compare the behavior of shock waves during their propagation in bubbly fluids with a viscous Newtonian carrier phase and a non-Newtonian carrier phase. We will make such an analysis using glycerin and a water solution of the polymer as an example, the polymer concentration being chosen such that the initial viscosity of the solution coincides with that of glycerin. In Fig. 1, profiles (d) and (e) illustrate the evolution of the shock wave with intensity $p_e = 0.15$ MPa, respectively, in the water solution of the polymer ($\rho_{10}^0 = 998$ kg/m^3, $\Sigma = 0.073$ kg/sec^2, $\eta_0 = 1.48$ Pa · sec, $\eta_s = 1$ mPa · sec, and $t_r = 1$ msec) and in the Newtonian fluid (glycerin, $\rho_{10}^0 = 1260$ kg/m^3 and $\eta_0 = 1.48$ Pa · sec) with air bubbles ($a_0 = 1$ mm, $\alpha_{20} = 0.02$, $T_0 = 293$, and $p_0 = 0.1$ MPa). It is seen that the behavior of the shock wave in these cases is fundamentally different: while the wave is of a monotone structure in the Newtonian fluid, the wave structure in the non-Newtonian fluid remains oscillatory. This is associated with the fact that during bubble oscillations in a non-Newtonian fluid

its effective viscosity can becomes considerably lower than the initial one. Note that the velocity of the shock wave in the first case is smaller, and this is explained by the fact that the glycerin density is much higher than that of the polymer solution.

For better insight into the processes occurring upon oscillations of the bubbles in a non-Newtonian fluid, we analyze the dynamic characteristics of a two-phase system. For stresses in a polymer and in a solvent, one can derive, from Eqs. (6) and (7), relations for complex amplitudes of these quantities in the case of harmonic oscillations of bubbles with a small real amplitude Δa according to the law

$$a = a_0 + \Delta a \exp(i\omega t), \qquad \Delta a \ll a_0. \tag{13}$$

For this purpose, we search for the stresses S_p and S_s in the form

$$S_k = S_k^* \exp(i\omega t), \qquad k = p, s. \tag{14}$$

For the velocity of the bubble wall w, we obtain

$$w = i\omega \Delta a \exp(i\omega t), \quad \text{i.e., the amplitude is} \quad w^* = i\omega \Delta a; \tag{15}$$

$$\frac{w}{a} = \frac{i\omega \Delta a \exp(i\omega t)}{a_0 + \Delta a \exp(i\omega t)} \approx i\omega \frac{\Delta a}{a_0} \exp(i\omega t), \quad \text{i.e.,} \quad \left(\frac{w}{a}\right)^* \approx i\omega \frac{\Delta a}{a_0}. \tag{16}$$

Having substituted (13)–(16) into (7), for the amplitude of stresses in a polymer, we write the relation

$$S_p^* = (-4\eta_p \Delta a/a_0)(i\omega/(1 + i\omega t_r)). \tag{17}$$

Similarly, for a solvent, we find from (6)

$$S_s^* = -4\eta_s i\omega \Delta a/a_0. \tag{18}$$

The amplitude of the total stress is as follows:

$$S^* = S_p^* + S_s^* = (-4i\omega \Delta a/a_0)(\eta_p/(1 + i\omega t_r) + \eta_s). \tag{19}$$

The real and imaginary components S_p^* ($S_p^* = S_p' + iS_p''$) are of the form

$$S_p' = -4 \frac{\eta_p}{t_r} \frac{\Delta a}{a_0} \frac{(t_r\omega)^2}{1 + (t_r\omega)^2}; \tag{20}$$

$$S_p'' = -4\eta_p \frac{\omega \Delta a}{a_0} \frac{1}{1 + (t_r\omega)^2}. \tag{21}$$

Their ratio is $S_p'/S_p'' = t_r\omega \sim t_r/t_*$, i.e., for $t_r \ll t_*$ ($t_r\omega \ll 1$), the imaginary component $S_p'' \approx -4\eta_p\omega\Delta a/a_0$ dominates, whereas, for $t_r \gg t_*$ ($t_r\omega \gg 1$), the real component $S_p' \approx -4\eta_p\Delta a/(t_r a_0)$ is dominant. As one should expect, these relations coincide with those obtained in the limiting cases considered for S_p (9) and (11), with the first corresponding to the viscous force and the second corresponding to the elastic one.

By analogy with the complex viscosity for small shear oscillations of non-Newtonian fluids [9], one can introduce the coefficient of complex dynamic viscosity upon small oscillations of a bubble in such fluids. In the Rayleigh–Lamb equation, the viscosity of a Newtonian fluid η_N is taken into account by the term $S_N = -4\eta_N w/a$, and the corresponding complex amplitude is $S_N^* = -4\eta_N i\omega\Delta a/a_0$, i.e., $\eta_N = S_N^*/(-4i\omega\Delta a/a_0)$.

We introduce the complex dynamic viscosity for pulsations of a bubble in a non-Newtonian fluid as in the last relation:

$$\eta^* = \eta' - i\eta'' = S^*/(-4i\omega\Delta a/a_0).$$

Here η' is the real component of the complex viscosity, which corresponds to the viscous force, and η'' is the imaginary component, which corresponds to the elastic force.

Note that the velocity of longitudinal deformation of an incompressible fluid in the radial flow about the bubble is $e_{rr} = \partial v/\partial r = -2v/r$, and its complex amplitude is $e_{rr}^*\big|_{r=a} = -2i\omega\Delta a/a_0$ by virtue of (16). Thus, $\eta^* = (\tau^{(rr)*}/(2e_{rr}^*))\big|_{r=a}$, i.e., such a definition of the complex viscosity is natural.
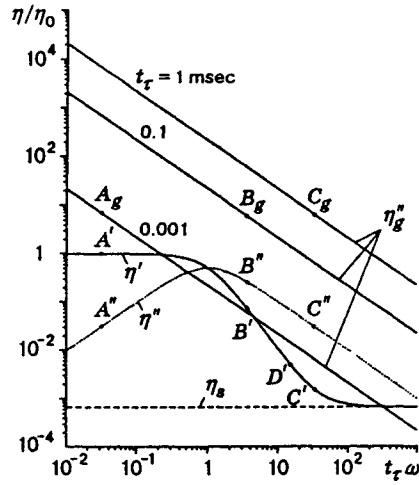
Fig. 2

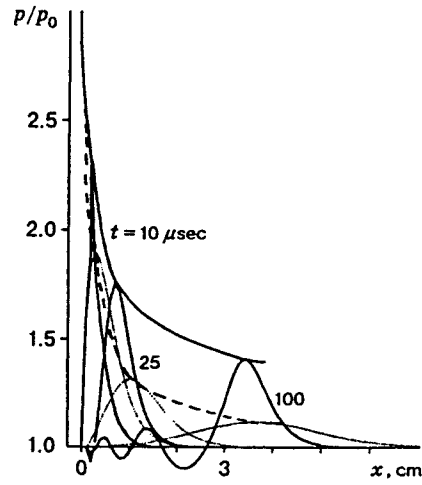Fig. 3

For a polymer fluid with one relaxation time, from (18)–(21) we have

$$\eta^* = \frac{\eta_p}{1 + it_r\omega} + \eta_s, \qquad \eta' = \frac{\eta_p}{1 + (t_r\omega)^2} + \eta_s, \qquad \eta'' = \frac{\eta_p t_r\omega}{1 + (t_r\omega)^2}. \tag{22}$$

These relations coincide with those derived for the components of the complex dynamic viscosity upon shear oscillations in a Maxwellian fluid with one relaxation time, except for an additive to the real component corresponding to the viscosity of the solvent.

Figure 2 shows the logarithmic dependences of the dynamic viscosity components η' and η'' on $t_r\omega$ for $\eta_s/\eta_0 = 0.001/1.48$. It is seen that the real component of the dynamic viscosity η' decreases monotonically from the initial η_0 to the viscosity of the solvent η_s with increasing $t_r\omega$, approaching the horizontal asymptote $\eta = \eta_s$ proportionally to $1/(t_r\omega)^2$.

In the η' curve, the point D' refers to the viscosity value that is realized during bubble oscillations in a polymer solution in a shock wave [profile (d) in Fig. 1]. This value is two orders of magnitude smaller than the initial viscosity, which leads to the appearance of oscillations in the shock wave propagating in a bubbly fluid with a non-Newtonian carrier phase, unlike the Newtonian fluid with viscosity η_0 [see profile (e) in Fig. 1].

The diagram of the viscosity η' also allows one to interpret the results of the calculations presented in Fig. 1. The points A', B', and C' in the curve for η' correspond to the relaxation times and oscillation frequencies of the bubbles for profiles (a)–(c) in Fig. 1. Clearly, for small relaxation times (point A'), a viscosity that is close to the initial one is realized, and the wave structure coincides with that in a Newtonian fluid with viscosity η_0. For large relaxation times (point C'), the viscosity of the Newtonian fluid is close to that of the solvent η_s, and the wave structure is the same as that in the Newtonian fluid with viscosity η_s. To profile (b) in Fig. 1 corresponds the point B', but since this case is rheologically nonlinear and the corresponding effective viscosity η' lies between η_0 and η_s, the wave profile in a mixture with a Newtonian carrier phase whose viscosity is the same η' does not coincide with profile (b).

The imaginary component of the complex dynamic viscosity η'' first increases proportionally to $t_r\omega$ from zero to maximum $\eta'' = \eta_p/2$ for $t_r\omega = 1$ and then decreases, approaching zero as $1/(t_r\omega)$. It has no effect on the wave structure, because the dynamic elasticity modulus $G' = \omega\eta''$ ($G^* \equiv G' + iG'' = i\omega\eta^*$), which corresponds to it, is substantially lower than the reduced elasticity modulus of the gas in the bubbles G_g (12). The latter modulus versus $t_r\omega$ is shown in Fig. 2 using the imaginary component of the reduced complex viscosity

$$\eta_g''(t_r\omega) = \frac{G_g}{\omega} = \frac{3}{4}\gamma_2 p_e t_r/(t_r\omega)$$

for $t_r = 0.001$, 01, and 1 msec.

690

As an illustration, the points A'', B'', and C'' in the curve for η'' of Fig. 2 pertain to the values of the imaginary dynamic viscosity of the fluid, which corresponds to the relaxation times and oscillation frequencies of the bubbles for profiles (a)–(c) in Fig. 1. The points A_g, B_g, and C_g show the corresponding values of η''_g and, as is seen in Fig. 2, are considerably above the points A'', B'', and C''.

It is evident from the diagrams considered above and formula (22) that within the framework of the Maxwell model, the frequency dependences of the dynamic moduli (viscosities) upon radial pulsations of the bubbles are of the same shapes as in the periodic shear. Thus, for an approximate analysis and approximation of the parameters of bubble pulsations in real polymer solutions, one can use the data on the spectra of dynamics moduli or viscosities that were obtained by conventional methods in a periodic shear.

We shall dwell upon the character of damping of nonlinear compression moments. The results of a numerical study are illustrated by compression waves whose initial shape is sinusoidal. Figure 3 shows the pattern of damping of such waves ($p_e = 0.3$ MPa and the initial pulse duration is 10 μsec) in a non-Newtonian fluid (the solid curve for the polymer solution) and in a Newtonian one (the dashed curve for glycerin) with air bubbles ($t_r = 0.1$ msec, and the remaining parameters are the same as in Fig. 1). It is seen that in the Newtonian bubbly fluid, the damping is less intense. This is connected with the fact that the intense wave damping occurs because of the high viscosity of the fluid, and the viscosity of the Newtonian fluid that is realized in the process becomes, as noted above, less than the initial one, i.e., less than the viscosity of glycerin.

The study of the effect of the determining parameters of a two-phase mixture (volume concentration of the bubbles, their radius, kind of gas, initial viscosity of the carrier phase and its density, time of stress relaxation, wave intensity, etc.) on the shock-wave evolution has shown that an increase in the relaxation time gives rise to an increase in the amplitude of oscillations in the shock wave and to a decrease in the intensity of damping of the compression moment. An increase in the initial viscosity of a polymer solution leads to a decrease in the amplitude of oscillations in the shock wave and to a more intense damping of the impact moment: with a rather high initial viscosity, the wave structure becomes monotone.

In the case of a bubbly fluid with a non-Newtonian carrier phase, the relationship between the dissipation that is due to interphase heat transfer and the dissipation that is due to the viscosity of a carrier fluid is a variable quantity and depends on the time of stress relaxation and on the initial fluid viscosity. Both kinds of dissipation can be a determining dissipation mechanism. Calculations performed for bubbles with various gases (air, carbon dioxide, helium, and sulfur hexofluorine) support this conclusion.

REFERENCES

1. A. A. Gubaidullin, A. I. Ivabdaev, R. I. Nigmatulin, and N. S. Khabeev, "Waves in a fluid with bubbles," *Itogi Nauki Tekh., Ser. Mekh. Zhidk. Gaza*, **17**, 160–249 (1982).

2. R. I. Nigmatulin, *Dynamics of Multiphase Media*, Part 2, Hemisphere Publ., New York (1991).

3. V. E. Nakoryakov, B. G. Pokusaev, and I. R. Shreiber, *Wave Dynamics of Gas- and Vapor-Liquid Media* [in Russian], Energoatomizdat, Moscow (1990).

4. A. A. Gubaidullin, A. I. Ivabdaev, and R. I. Nigmatulin, "Nonstationary waves in a fluid with gas bubbles," *Dokl. Akad. Nauk SSSR*, **226**, No. 6, 1299–1302 (1976).

5. S. P. Levitskii and A. T. Listrov, "Small oscillations of a gas-filled spherical cavity in viscoelastic polymer media," *Prikl. Mekh. Tekh. Fiz.*, No. 1, 137–142 (1974).

6. S. P. Levitskii and Z. P. Shul'man, *Dynamics and Heat and Mass Transfer of Bubbles in Polymer Fluids* [in Russian], Nauka Tekh., Minsk (1990).

7. G. Astarita and G. Marrucci, *Principles of Non-Newtonian Fluid Mechanics*, McGraw-Hill (1974).

8. C. J. S. Petrie, *Elongational Flows*, London (1979).

9. J. D. Ferry, *Viscoelastic Properties of Polymers*, John Wiley and Sons, New York, (1980).